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FEASIBILITY OF OPTIMIZING NOZZLE PERFORMANCE FOR ORBITAL-LAUNCH NUCLEAR ROCKETS

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SUMMARY

Nozzle performance for orbital-launch nuclear rockets was evaluated for a range of operating conditions and nozzle geometries. From this study, conclusions were drawn concerning (1) optimum values of nozzle area ratio and operating pressure, (2) operating conditions for favorable nozzle heat transfer, and (3) the effects of deviations from optimum conditions.

from p2 In general, the nozzle efficiency increases with both increasing divergence angle and increasing stagnation-pressure level. For a given set of conditions, however, the nozzle efficiency increases with increasing area ratio only until the gains in performance due to additional expansion are offset by the increase in convective heat transfer. The variation of specific impulse with pressure is maximized at a pressure level that depends upon the pressure at which the flow freezes. On the basis of heat transfer, operation on the low-pressure side of the optimum point is desirable. This mode of operation sacrifices very little in performance but permits a substantial reduction in the throat heat flux. end

INTRODUCTION

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A basic problem associated with nuclear rockets is the optimization of the energy absorbed by the hydrogen propellant as it passes through the reactor and, in turn, the maximization of the power available for thrust (so-called jet power). Of course, the amount of power that can be absorbed by the hydrogen depends upon the maximum reactor temperature and the propellant pressure. With the appropriate reactor operating conditions selected, nozzle operating parameters are desired that will maximize the jet power. In reference 1, it was demonstrated that moderate changes in reactor parameters do not change the reactor performance greatly from that obtained under optimum operating conditions; consequently, nozzle-inlet conditions may be varied somewhat without detracting from the reactor performance. Apparently, sizable dividends may be available by optimizing nozzle performance. The approach to be considered herein will be to maximize the jet power and then to determine the resulting specific impulse and thrust.

At the high temperature levels being considered (~5000° R) for nuclear reactors, two major nozzle problems are encountered. These are the dissociation of the hydrogen and the heat transfer to the nozzle walls. Both of these problems are intimately related to the temperature and the pressure of the propellant. If

the flow is assumed to be in thermodynamic equilibrium, a reduction in operating pressure at constant temperature increases the enthalpy level and consequently increases the specific impulse. Thus, with this assumption, it is advantageous to operate at a low pressure, which, incidentally, is a design variable subject to the designer's choice. On the other hand, if the flow is assumed to be frozen, it is desirable to operate at a high pressure so that dissociation is inhibited and the amount of energy frozen or invested in dissociation is a minimum. Actually, however, the nozzle flow process will be a nonequilibrium process, so that the effect of pressure on nozzle performance is not well defined and has to be considered quite carefully in any analysis.

The need for considering the heat transferred to the nozzle walls in a nozzle study is well exemplified by the studies of reference 2. The heat-transfer analysis of reference 2 shows that, even though the hydrogen propellant has a large heat capacity, a nuclear-rocket nozzle cannot be regeneratively cooled for the assumed conditions because the throat heat fluxes encountered are so high. Again, the indications are that operation at a low pressure level is desirable since the throat heat flux would be low. This trend, however, cannot be fully accepted until all important nozzle parameters are considered together and worked into a nozzle optimization study.

The intent of this report is to make a general nozzle analysis to determine the effect of frozen-flow losses, heat-transfer losses, and expansion losses on the performance characteristics of a nuclear-rocket nozzle. In particular, nozzle operating conditions and geometry will be so varied that tentative conclusions concerning optimum operating pressure and nozzle geometry may be reached, with the hope that better performance may be achieved while the convective heat transfer is reduced.

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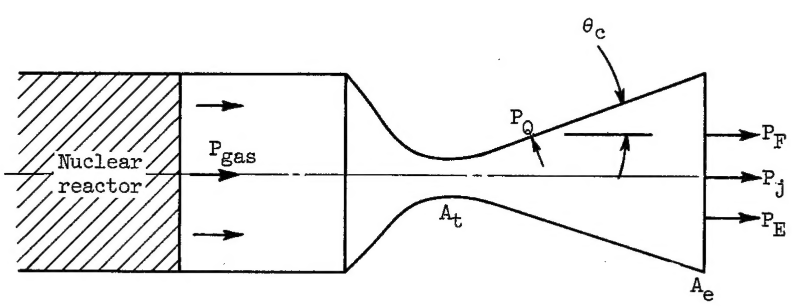
ANALYSIS

The problem to be considered in this section may be stated as follows: The hydrogen propellant, upon leaving the nuclear reactor, has associated with it a given temperature level and a certain amount of power. What nozzle geometry and operating pressure must be chosen to optimize the power available for thrust (jet power)? The problem may be defined with a nozzle power balance.

Power Balance

A hypothetical nuclear-rocket nozzle is shown in the following sketch with the contributing terms of the power balance identified:



(All symbols are conveniently grouped and defined in the appendix.) The propellant (or gas) power P_{gas} must be divided between the convective power loss P_Q, the frozen power loss P_F, the expansion power loss P_E, and the jet power P_j. Consequently, the nozzle power balance is given by

$$P_{gas} = P_j + P_F + P_Q + P_E \tag{1}$$

An overall nozzle efficiency may be obtained from equation (1) by forming the ratio of jet power to gas power:

$$\eta = \frac{P_j}{P_{gas}} = 1 - \frac{P_F + P_Q + P_E}{P_{gas}} \tag{2}$$

Equation (2) focuses attention upon the fact that the various nozzle losses must be kept to a minimum in order to achieve maximum nozzle performance. If a regenerative cooling system is employed to minimize the convective power loss P_Q, the reactor power required to yield a given gas power P_{gas} is less by the amount of power recovered regeneratively.

Equation (2) may be cast into a more convenient form involving a frozen-flow efficiency η_F, a convective-heat-transfer efficiency η_Q, and an expansion efficiency η_E:

$$\eta = \eta_F \eta_Q \eta_E \tag{3}$$

where the respective efficiencies are given by

$$\eta_F = \frac{P_{gas} - P_F}{P_{gas}} \tag{4}$$

$$\eta_Q = \frac{(P_{gas} - P_F) - P_Q}{P_{gas} - P_F} \tag{5}$$

$$\eta_E = \frac{(P_{gas} - P_F - P_Q) - P_E}{P_{gas} - P_F - P_Q} = \frac{P_j}{P_{gas} - P_F - P_Q} \tag{6}$$

The problem that now remains is to find P_F, P_Q, and P_E.

Frozen Power Loss

During the expansion of the hydrogen (H_2) through the rocket nozzle, atomic hydrogen (H) recombines to some extent and thereby releases dissociation energy and increases the nozzle performance above that to be expected without recombination (i.e., the frozen-flow case). In such a flow situation, the performance gains to be made through recombination depend to a great extent on how long local

chemical equilibrium may be maintained. Some criterion must thus be employed to predict the departure of the nozzle flow from chemical equilibrium. For this purpose, the near-equilibrium flow criterion proposed in reference 3 for rocket-nozzle performance calculations will be used. Specifically, this method permits an estimate to be made of a small temperature difference representative of the difference between the equilibrium and the nonequilibrium reactant weight fractions.

The reaction of interest is

$$2H + M \underset{k_r}{\overset{k_d}{\rightleftarrows}} H_2 + M$$

where the third body M could be either H_2 or H. The relation of reference 3 for this reaction is given by

$$T' - T \approx -\frac{dT}{dt}\left[k_r\left(\frac{p}{RT}\right)^2 X_H^2 X_M\left(\frac{4}{X_H} + \frac{1}{X_{H_2}}\right)\right]^{-1} \tag{7}$$

where

k_d dissociation rate, liters^2/(mole^2)(sec)

k_r recombination rate, liters^2/(mole^2)(sec)

p pressure, atm

R universal gas constant, (liters)(atm)/(^{o}K)(mole)

T static temperature, ^{o}K

T' static temperature relating actual concentrations to equilibrium values, ^{o}K

$\frac{dT}{dt}$ rate of change of temperature in nozzle, ^{o}K/sec

X_i mole fraction of constituent i

Rearranging equation (7) and assuming, as in reference 3, that $X_H \leq 0.1$ and $T' - T \approx 20^{o}$ K yield the following equation for the pressure:

$$p^2 = \frac{2.5\left(-\frac{dT}{dt}\right)(RT)^2}{20k_r}, \text{ atm}^2 \tag{8}$$

For the nuclear-rocket-nozzle problem, the following typical parameters have been chosen to illustrate the approximate pressure level required to assure equilibrium flow:

$$k_r \approx 10^{11} \text{ liters}^2/(\text{mole}^2)(\text{sec})$$

$$T \approx 2800^\circ \text{ K}$$

$$-\frac{dT}{dt} \approx 10^{6\circ} \text{ K/sec}$$

Substitution of these parameters into equation (8) yields $p \geq 1/3$ atmosphere for equilibrium flow. Actually the term $-dT/dt$ varies inversely with nozzle length; considerable latitude in the pressure required to assume equilibrium may thus be obtained by varying the nozzle length. The use of the criterion of reference 3 does not imply that it will predict the point of departure from equilibrium accurately. On the contrary, at best, it will indicate only the general region of the flow where departures from equilibrium are likely. For the purposes of this report, however, the method should be adequate to demonstrate the feasibility of optimizing nozzle performance.

With the freezing pressure and stagnation conditions known, the nozzle pressure ratio at which freezing occurs can be found and with the aid of table II of reference 4 the degree of dissociation α can be calculated from the following equation:

$$\alpha = \frac{X_H}{2 - X_H} \tag{9}$$

Since α has been determined, the frozen power loss may be determined from

$$P_F = 1.055 \, \dot{w}H_d, \text{ kw} \tag{10}$$

where

$$H_d = \frac{41{,}526\alpha E_i}{M}, \text{ Btu/lb} \tag{11}$$

Convective-Heat-Transfer Power Loss

A nuclear-rocket nozzle is heated in three different ways: (1) convective heat transfer from the propellant, (2) gamma heating from the reactor, and (3) thermal radiation from the reactor face. All these heat losses must be considered in determining the total nozzle heat load (i.e., the total amount of heat to be carried away by a cooling system) but need not be considered in defining nozzle performance. The only losses that must be considered in performance calculations are those that stem from the propellant, that is, those losses that diminish the propellant power P_{gas}. Consequently, the heat loss of interest herein is the loss associated with the convective heat transfer from the propellant to the nozzle walls.

No precise calculating procedure (i.e., one that accounts for dissociation, axial pressure, and temperature gradients) exists for predicting nozzle heat transfer. A straightforward, approximate approach that has been used successfully in the past is therefore employed. The relation used to predict the local

heat-transfer coefficient h is that suggested by reference 5 for fully developed turbulent pipe flow:

$$h = \frac{0.023}{d^{0.2}} \frac{(0.067\mu_{ref})^{0.2}}{Pr_{ref}^{2/3}} \left(\frac{\dot{w}}{A}\right)^{0.8} \left(\frac{T}{T_{ref}}\right)^{0.8}, \text{ lb/(sq ft)(sec)} \tag{12}$$

where μ_{ref} and Pr_{ref} are evaluated at a reference temperature T_{ref} defined as

$$T_{ref} = \frac{1}{2}(T + T_w) \tag{13}$$

The convective heat flux may then be obtained from

$$q = h(H_o - H_w), \text{ Btu/(sq ft)(sec)} \tag{14}$$

The total nozzle heat loss that is required for evaluating nozzle performance may be estimated conveniently by the method of reference 6, in which the total convective power loss to a conical nozzle in terms of the local heat flux at the nozzle throat is given by

$$P_Q = 1.055\, q_t A_t \left(0.8 + \frac{\ln \frac{A_e}{A_t}}{\sin \theta_c}\right), \text{ kw} \tag{15}$$

The use of equation (15) simplifies the calculation considerably since only the throat heat flux need be determined. The remaining parameters are specified through the nozzle geometry.

Expansion Power Loss

The expansion power loss is determined by the amount of thermal energy remaining in the propellant flow at the exit plane of the nozzle. This power loss is thus directly related to the expansion area ratio; the greater the expansion area ratio, the smaller the expansion power loss. There is no restriction on the expansion area ratio due to ambient pressure as this pressure is taken equal to zero in orbit.

Calculations of these losses assume a one-dimensional isentropic expansion of the propellant through the nozzle. In reality, the isentropic assumption is in conflict with the previous discussion concerning convective-heat-transfer losses. Even so, reference 3 indicates that heat transfer to the nozzle walls will introduce only small errors into the calculated performance for rocket engines of reasonable size. The expansion power loss is found from the following equation:

$$P_E = 1.055\, \dot{w} H_E, \text{ kw} \tag{16}$$

where H_E is determined by nozzle stagnation conditions and nozzle expansion area ratio.

Optimization Procedure

The performance calculations upon which the optimizations and comparisons are based were made by specifying the following basic initial parameters: reactor-outlet temperature T_o, reactor power level P_{gas}, and reactor-outlet pressure level p_o. The variables to be optimized are expansion area ratio A_e/A_t, nozzle efficiency η or jet power P_j, and finally the stagnation pressure p_o.

The method will now be outlined briefly. With the reactor-outlet temperature T_o and a selected stagnation pressure p_o, the stagnation-enthalpy level H_o can be determined from reference 4. Now, with the selected operating conditions and the criterion of reference 3, the approximate pressure at which the flow freezes can be found. This pressure then determines the nozzle area ratio (see ref. 4) at which the flow freezes and permits α to be determined (eq. (9)). Once the "freezing" area ratio is known, the static-enthalpy distribution through the nozzle may be calculated. With this information, the frozen power loss may be found from equation (10), and the expansion power loss at any desired exit area ratio may be determined from equation (16).

All that remains to be found in order to determine nozzle performance is the power invested in convective heat transfer P_Q. Finding this power loss requires the nozzle flow rate and the throat area. The nozzle flow rate is given by the reactor power level and the stagnation enthalpy:

$$\dot{w} = \frac{P_{gas}}{1.055\ H_o},\ \text{lb/sec} \tag{17}$$

If the flow through the nozzle throat is in equilibrium, as determined by the criterion of reference 3, the throat area is found by calculating an isentropic expansion from the stagnation conditions and maximizing the product ρu. The throat area is then given by

$$A_t = \frac{\dot{w}}{\rho u},\ \text{sq ft} \tag{18}$$

For the case in which the flow through the throat is frozen, the throat area is determined from the perfect-gas equation

$$A_t = \sqrt{\frac{R}{g}}\,\frac{\dot{w}}{p_o}\sqrt{\frac{ZT_o}{\frac{2\gamma}{\gamma+1}\left(\frac{2}{\gamma+1}\right)^{2/(\gamma-1)}}},\ \text{sq ft} \tag{19}$$

where $Z = 1 + \alpha$ has been inserted to account for compressibility and both α and γ are determined by the conditions at the freezing area ratio. With this information, the convective power loss P_Q is readily found from equations (12) to (15).

After all the powers required to determine performance have been found, the specific impulse and the thrust may be found from the jet power P_j as follows:

$$P_j = \eta P_{gas}$$

and

$$\left.\begin{aligned} I &= \sqrt{\frac{45.9\ \eta P_{gas}}{\dot{w}}},\ \text{sec} \\ F &= \dot{w} I,\ \text{lb} \end{aligned}\right\} \tag{20}$$

Equations (20) neglect the pressure-area term in the momentum equation; this is justified, however, when it is recalled that the ambient pressure is taken equal to zero in a space orbit and that the nozzle-exit Mach number is large, that is, the nozzle-exit static pressure is quite small.

By use of the procedure just outlined and the assumptions and relations described in the previous sections, nozzle efficiencies were calculated for the desired combination of independent variables and the maximum nozzle efficiencies were found.

The following values and ranges of variables were used:

Reactor-outlet temperature, T_o, ^{o}R	5040
Reactor-outlet pressure, p_o, atm	0.01 to 100
Reactor power level, P_{gas}, kw	7000
Conical nozzle half-angles, θ_c, deg	10, 15, 20, 25
Nozzle wall temperature, T_w, ^{o}R	1440

These parameters are typical of those chosen for the low-power space nuclear rocket discussed in reference 7.

RESULTS AND DISCUSSION

The primary purpose of this analysis was to determine the nozzle operating conditions and the geometry that would produce the optimum nozzle performance. Emphasis was also placed on the total convective heat load and the nozzle-throat heat flux, since both may constitute a major design limitation.

As might be surmised from the ANALYSIS, the effect of freezing pressure upon nozzle performance is quite important. For a better understanding of the nozzle problem, therefore, the discussion is initiated with consideration of the general effect of freezing pressure. Then, for a detailed discussion of nozzle performance, a specific but realistic freezing pressure (based on the best data available) is chosen to make complete nozzle performance calculations.

Effect of Freezing-Pressure Level

According to the near-equilibrium criterion of reference 3, a local nozzle pressure greater than, or equal to, a certain value (p_{fr}) must be obtained in order for equilibrium flow to exist. If the local pressure falls below p_{fr}, the flow departs from equilibrium, approaches a frozen condition in a short transitional region, and finally is completely frozen. For analysis purposes, the flow will be assumed to freeze suddenly at p_{fr}. Thus, if the nozzle-inlet pressure is greater than p_{fr}, the flow will be in equilibrium in the nozzle until it expands to a pressure less than or equal to p_{fr}. The expansion from this point on is then completely frozen. If the nozzle-inlet pressure is less than p_{fr}, the flow is completely frozen. The pressure at which the flow freezes should thus affect the nozzle performance considerably. As the inlet pressure increases and exceeds the freezing pressure, the overall nozzle efficiency should depart from the frozen curve and approach the equilibrium curve.

The effect of freezing pressure may be observed in figure 1 for a nozzle half-angle θ_c of 15°, an area ratio A_e/A_t of 50, and for three freezing pressures: <0.01, 1/3, and 100 atmospheres. For some freezing pressure less than 0.01 atmosphere, the nozzle flow will be in complete equilibrium for the inlet pressures considered. (This curve is presented as a reference curve and represents the best achievable performance for the conditions investigated.) For a p_{fr} of 100 atmospheres, the flow will be completely frozen, since all inlet pressures considered are less than 100 atmospheres and consequently all nozzle pressures will also be less than 100 atmospheres. Also presented in figure 1 is the nozzle-efficiency variation for the freezing pressure (1/3 atm) derived with the aid of equation (8). As the pressure increases and exceeds 1/3 atmosphere, the flow departs from the completely frozen curve and the nozzle efficiency approaches the equilibrium efficiency. The equilibrium curve is reached at a pressure level of approximately 3 atmospheres, and then the two curves are the same. As will be shown later, operation at a low pressure has the advantage of decreasing the throat heat flux greatly, which thereby helps to alleviate a heat-transfer problem. On the other hand, the total heat load increases; however, this increase poses no real design problem. All the following results to be presented are based on the value of p_{fr} of 1/3 atmosphere, which was derived in the ANALYSIS.

Effect of Expansion Area Ratio

The effect of nozzle expansion area ratio on overall nozzle efficiency for a given stagnation pressure and divergence angle is shown in figure 2 for stagnation pressures of 0.01, 1, and 100 atmospheres. In general, the nozzle efficiency increases as the expansion area ratio increases. A point is finally reached, however, at which an additional increase in area ratio decreases the nozzle efficiency. At this point, the gain in performance from additional expansion is offset by the increase in nozzle convective heat loss. In addition, figure 2 indicates that the nozzle efficiency increases with increasing nozzle divergence angle θ_c and that the optimum efficiency at a given θ_c progresses

to higher area ratios as the divergence angle increases. Even though maximum nozzle efficiencies have been found, the penalty associated with moderate deviations from the maximum is not large, since each curve is rather flat in the region of the maximum efficiency. An area ratio less than optimum could therefore be chosen to reduce nozzle weight.

Effect of Stagnation-Pressure Level

The variation of maximum nozzle efficiency from figure 2 with stagnation-pressure level is illustrated in figure 3. For a given divergence angle, an increase in stagnation pressure produces a substantial increase in nozzle efficiency. A stagnation-pressure level is finally reached, however, after which an increase in pressure produces a very small increase in nozzle efficiency. Thus, for the case being considered, gains in performance are negligible for stagnation pressures greater than about 10 times the freezing pressure. The same behavior pattern occurs for all divergence angles, the only difference being that the absolute level for the nozzle-efficiency curve increases as the divergence angle increases. At a pressure of 3 atmospheres, changing the divergence angle from 10° to 25°, for example, changes the maximum nozzle efficiency from 0.85 to 0.92.

The change in thrust level at optimum nozzle efficiency with stagnation pressure is shown in figure 4. The preceding discussion concerning nozzle efficiency is directly applicable. The only additional point to be made is that the optimum thrust level does not change much with nozzle divergence angle.

The most interesting effect of stagnation-pressure level may be found in figure 5, in which the variation of specific impulse with pressure at maximum nozzle efficiency is presented for divergence angles of 10° and 25°. The specific impulse is optimized at a pressure level of approximately 3 atmospheres for both divergence angles. This result, of course, is a consequence of the freezing pressure used in the analysis. The optimum pressure level would vary with the freezing pressure.

The previous discussion has been concerned only with the optimum parameters. This approach is not too realistic because the optimum area ratio increases greatly as the stagnation pressure increases. At a stagnation pressure of 100 atmospheres, for example, the optimum area ratio for a divergence angle of 25° is of the order of 20,000. It is therefore of interest to consider more practical area ratios, as in figure 6, in which specific impulse is given in terms of stagnation pressure and area ratio for a divergence angle of 15°. Considering area ratios other than the optimum does not change the character of the curves. Reducing the area ratio, however, slightly reduces the maximum specific impulse obtainable. At a stagnation pressure of 3 atmospheres, for example, the optimum specific impulse is only 1.06 times that obtained for an area ratio of 25. Apparently, area ratios varying all the way from the optimum value to 25 may be employed without a significant decrease in specific impulse (although nozzle efficiency decreases considerably).

Effect of Heat Transfer

As noted in figure 5, the maximum specific impulse is not much greater than those obtained at pressures greater than or slightly less than 3 atmospheres.

This fact is significant in terms of the nozzle heat-transfer problem for the following reasons: The feasibility of nozzle cooling for a given set of conditions is primarily determined by the throat heat flux, since it is the highest heat flux experienced by a nozzle. If regenerative cooling is employed, the coolant flow must be capable of handling the highest nozzle heat flux, namely, the throat heat flux. This requirement is rather severe, as noted in reference 2, in which this particular aspect of regenerative cooling was investigated. If radiation cooling is employed, the local heat flux must be radiated away at a nozzle wall temperature that does not exceed the nozzle-wall-material melting temperature. These considerations are the most important ones; a secondary effect, however, that must be considered for the case of regenerative cooling is the total cooling load. As noted in equation (15), this effect is governed also by the throat heat flux; however, since hydrogen has a good heat capacity, this effect is not so important as the local heat flux at the nozzle throat.

As the stagnation pressure is increased, the throat heat flux increases; in fact, it increases approximately as the 0.75 power of the stagnation pressure ($q_t = 329\ p_o^{0.75}$). It is therefore most desirable to operate at the lowest pressure possible without sacrificing greatly on performance. Figure 7 shows that by operating at the pressure level yielding the maximum specific impulse (3 atm), the throat heat flux may be reduced by over a factor of 10 from the value obtained at a pressure of 100 atmospheres. In addition, by operating at a pressure slightly lower than the optimum pressure, the specific impulse is not changed greatly, but the throat heat flux is changed considerably. Operation at 1 atmosphere, for example, reduces the specific impulse by approximately 4 percent and reduces the throat heat flux by a factor of 2 below that obtained at the optimum stagnation pressure. A small sacrifice in specific impulse thus reduces the throat heat flux by a large amount. Although the sacrifice in specific impulse is not large, reference to figure 3 shows that the optimum efficiency is reduced by approximately 13 percent, which requires a higher reactor power for a fixed thrust and specific impulse.

The effect of stagnation-pressure level on the total convective cooling load may be seen in figure 8 for a divergence angle of 15° and an area ratio of 50. Reducing the pressure increases the total convective heat load because the throat area increases faster than the throat heat flux decreases (see eq. (15)). This increase in Q, however, is slight and is not so important a consideration as the throat heat flux q_t.

At this point it should be reemphasized that the heat load presented in figure 8 does not represent the total heat load that a cooling system would have to handle. The heat due to gamma rays and thermal radiation from the face of the reactor is lacking. Estimates of these heat-transfer effects may be made, however. In reference 3, it is shown that the nuclear heating can be as much as 10 percent of the reactor power at a reactor power level of 50 megawatts. As the reactor power decreases, however, the nuclear heating also decreases. For the present case (7000 kw), it is therefore estimated that the nuclear heating is negligible compared with the other heating effects. The term that remains to be estimated is that due to thermal radiation. This term has been estimated by utilizing the data of reference 2, and its value is approximately 440 Btu per second. Since the thermal radiation effect is not a function of the pressure level, 440 Btu per second may be added to the curve in figure 8 to obtain an

estimate of the total heat load to be handled by a cooling system for the conditions investigated herein.

Effect of Power Level

The immediate effect of changing the power level for a given reactor-outlet temperature T_o, reactor-outlet pressure p_o, and wall temperature T_w is to vary the propellant flow rate $\dot{w}$. This change in flow rate, in turn, affects the nozzle performance through the convective heat transfer and the thrust.

The new flow rate $\dot{w}_2$ is related to the flow rate used in the analysis $\dot{w}_1$ by

$$\dot{w}_2 = \frac{P_{gas,2}}{P_{gas,1}} \dot{w}_1 = k\dot{w}_1 \tag{21}$$

With the use of equation (21) and

$$A_{t,2} = kA_{t,1} \tag{22}$$

the local throat heat flux and the total nozzle heat load are found from equations (14) and (15) to be

$$q_{t,2} = k^{-0.1}q_{t,1} \tag{23}$$

$$P_{Q,2} = k^{0.9}P_{Q,1} \tag{24}$$

The heat-transfer efficiency η_Q then becomes

$$\eta_{Q,2} = 1 - k^{-0.1}(1 - \eta_{Q,1}) \tag{25}$$

Investigation of the effect of k on $\eta_{Q,2}$ for typical values of $\eta_{Q,1}$ and for values of k ranging from 1 to 100 shows that $\eta_{Q,2}$ does not vary significantly from $\eta_{Q,1}$, at most about 6 percent. As the propellant power is increased, the specific impulse therefore remains almost constant (as would be expected with a constant stagnation temperature T_o) and the thrust increases directly as the power increases; that is, $F_2 = kF_1$. Thus, the maximum specific impulses found at a P_{gas} of 7000 kilowatts should also be approximately right for higher power levels.

The throat heat flux is reduced as the power level is increased. With $k = 100$, for example, the throat heat flux is 0.63 of the value obtained for a power level of 7000 kilowatts.

Effect of Nozzle-Wall Temperature

Since the nozzle-wall temperature has been considered an independent design variable, its effect upon the overall nozzle efficiency should be considered. This effect will not be treated in detail but in a general manner, and some general conclusions concerning it will be made.

As the wall temperature is increased, the driving potential for the local heat flux $H_O - H_W$ is reduced, and thereby the local heating rate is reduced. The net result is that the heat-transfer efficiency η_Q is increased with a corresponding increase in overall nozzle efficiency. The total amount of power transferred to a hot wall in terms of the power transferred to a cold wall is given approximately by

$$P_{Q,hot} = \frac{(H_O - H_W)_{hot}}{(H_O - H_W)_{cold}} P_{Q,cold}$$

This expression is not exact, of course, since the heat-transfer coefficient h would change slightly with wall temperature.

CONCLUSIONS

The feasibility of optimizing nozzle performance for an orbital-launch nuclear rocket was considered. Power losses due to dissociation, heat transfer from the propellant to the engine walls, and incomplete expansion of the propellant were evaluated. Operating conditions used (reactor power level, 7000 kw and reactor-outlet temperature, 5040° R) are those under consideration for low-power space nuclear rockets. In addition, a partly frozen nozzle flow was studied. The following conclusions based on the analysis presented herein may be made:

1. The nozzle efficiency for a given nozzle divergence angle and pressure increases with increasing area ratio until the gains in performance due to additional expansion are offset by the increase in convective heat transfer. At this area ratio, the nozzle efficiency starts to decrease. In general, the nozzle efficiency increases with both increasing divergence angle and stagnation-pressure level.

2. For a given divergence angle, the maximum nozzle efficiency increases with increasing stagnation-pressure level. A stagnation pressure is finally reached, however, at which the variation of nozzle efficiency with pressure is small. The optimum thrust level behaves in a similar manner with the stagnation-pressure level.

3. The specific impulse is maximized at a pressure level that depends upon the pressure at which the flow freezes. For the case treated herein, in which the flow freezes at a pressure of 1/3 atmosphere, the best stagnation-pressure level is about 10 times the freezing pressure. On the basis of heat transfer, operation on the low-pressure side of the optimum point is desirable. This mode

of operation sacrifices very little in specific impulse but causes a substantial reduction in the throat heat flux.

4. The throat heat flux increases substantially with increasing stagnation pressure and varies approximately as the 0.75 power of the stagnation pressure.

5. The total heat load to be handled by a cooling system increases with decreasing pressure.

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National Aeronautics and Space Administration
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APPENDIX - SYMBOLS

A	flow area, sq ft
d	diameter, ft
E_i	dissociation potential, v
F	thrust, lb
g	acceleration due to gravity, ft/sec^2
H	enthalpy, Btu/lb
I	effective specific impulse, sec
M	molecular weight, lb
P	power, kw
Pr	Prandtl number
p	pressure, atm
Q	total heat flux, Btu/sec
q	local heat flux, Btu/(sq ft)(sec)
R	hydrogen gas constant
T	temperature, $^{\circ}$R
t	time, sec
u	stream velocity, ft/sec
$\dot{w}$	propellant flow rate, lb/sec
α	degree of dissociation
γ	ratio of specific heats for frozen flow
η	nozzle efficiency
θ_c	conical nozzle half-angle (divergence angle), deg
μ	absolute viscosity, micropoises
ρ	stream density, lb/cu ft

Subscripts:

d	dissociation

E	expansion
e	nozzle exit
F	frozen
fr	freezing
gas	propellant
j	jet
o	reactor outlet (nozzle stagnation)
Q	heat transfer
ref	reference condition for heat transfer
t	nozzle throat
w	nozzle wall

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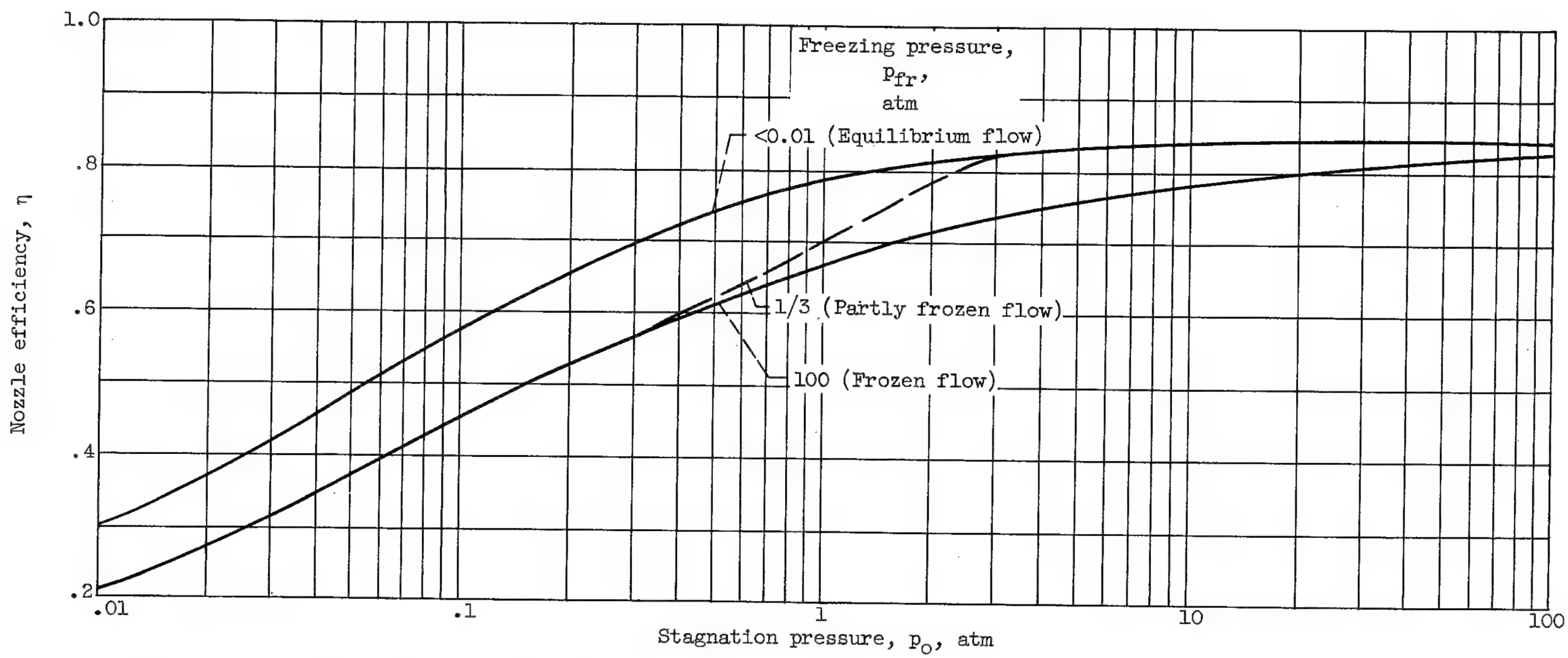
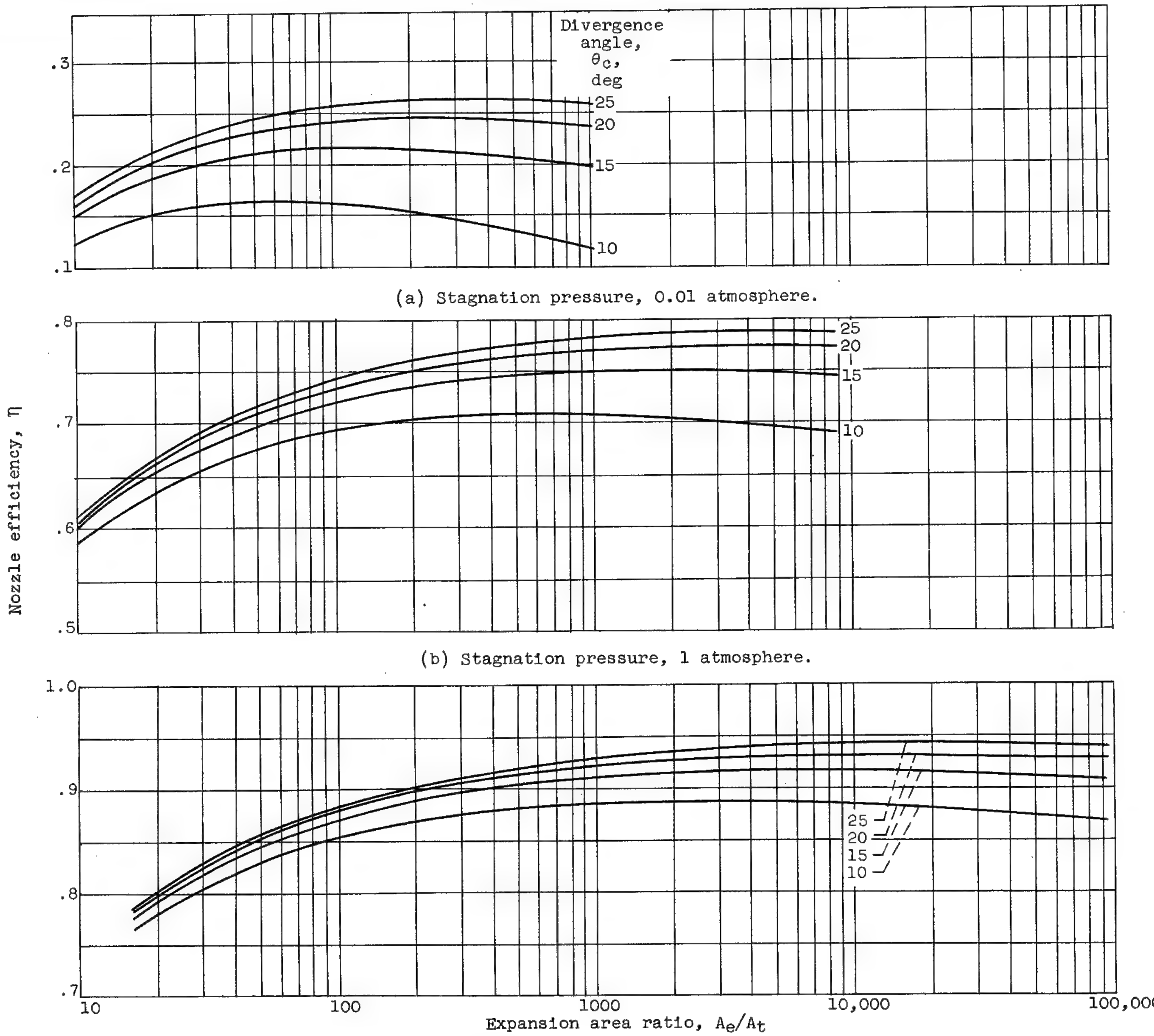


Figure 1. - Comparison of equilibrium, frozen, and partly frozen flows. Divergence angle, 15°; expansion area ratio, 50.



(a) Stagnation pressure, 0.01 atmosphere.

(b) Stagnation pressure, 1 atmosphere.

(c) Stagnation pressure, 100 atmospheres.

Figure 2. - Variation of nozzle efficiency with nozzle geometry.

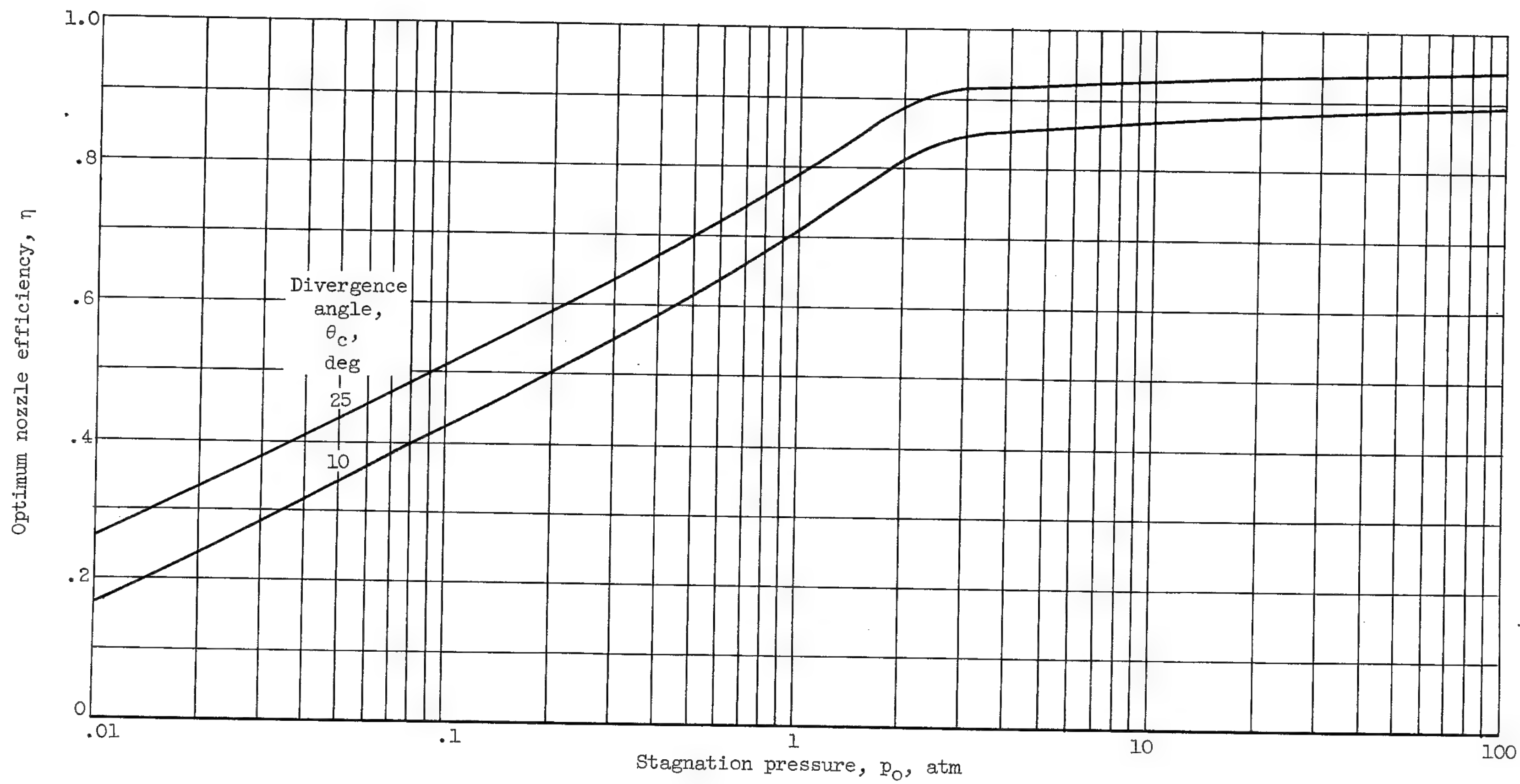


Figure 3. - Effect of stagnation pressure on optimum nozzle efficiency.

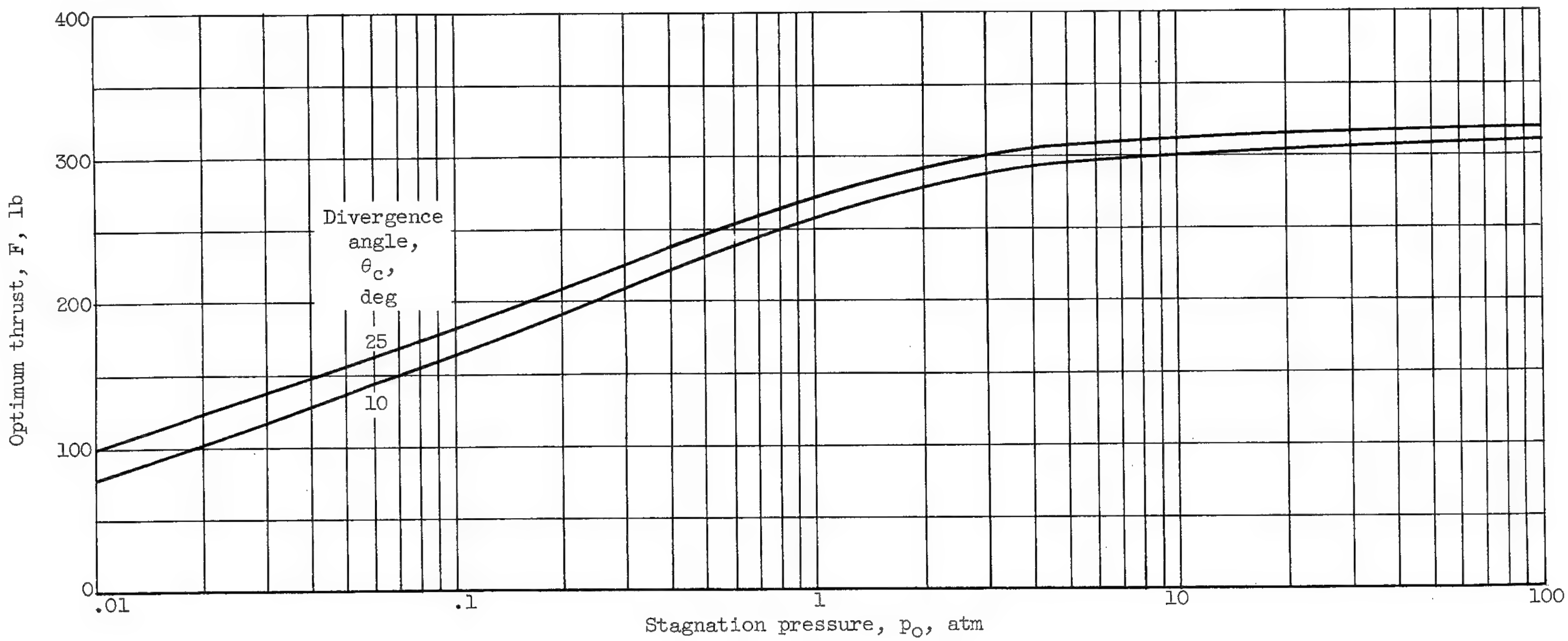


Figure 4. - Effect of stagnation pressure on optimum thrust.

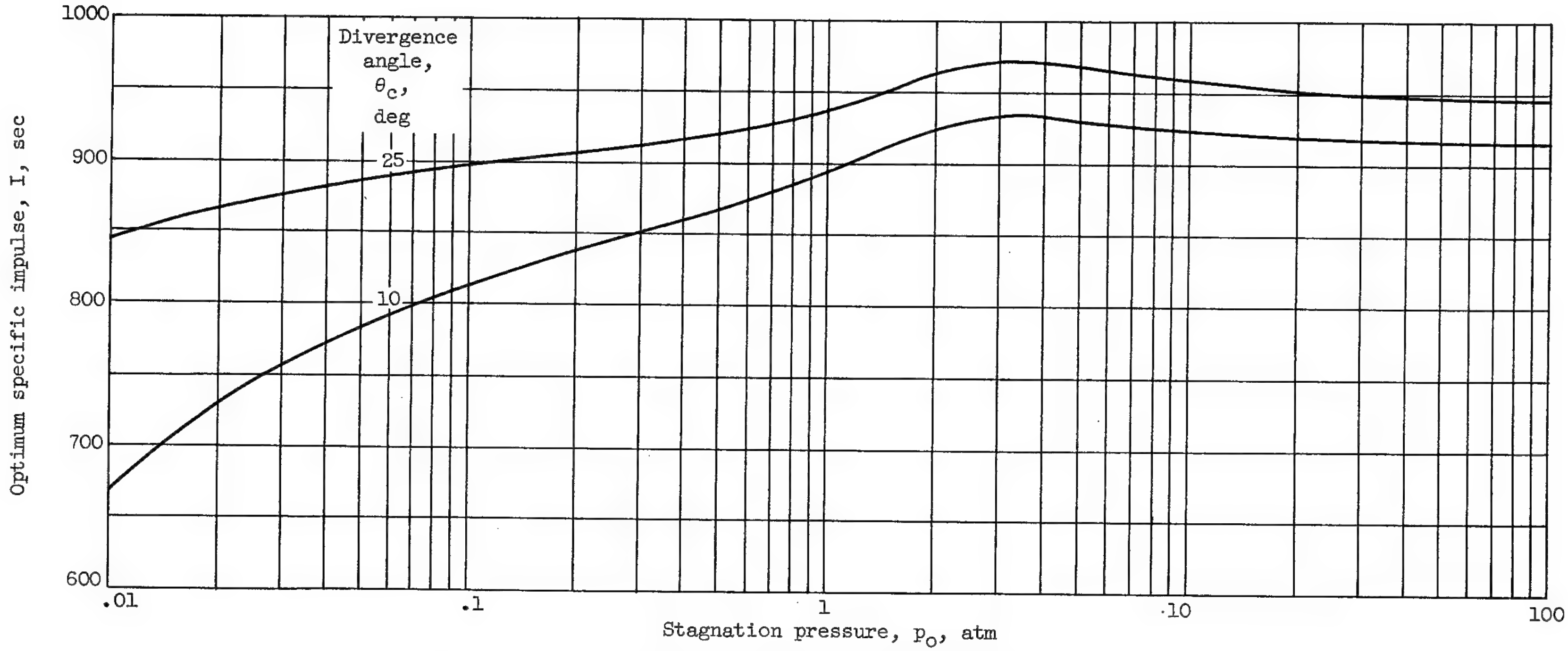


Figure 5. - Effect of stagnation pressure on optimum specific impulse.

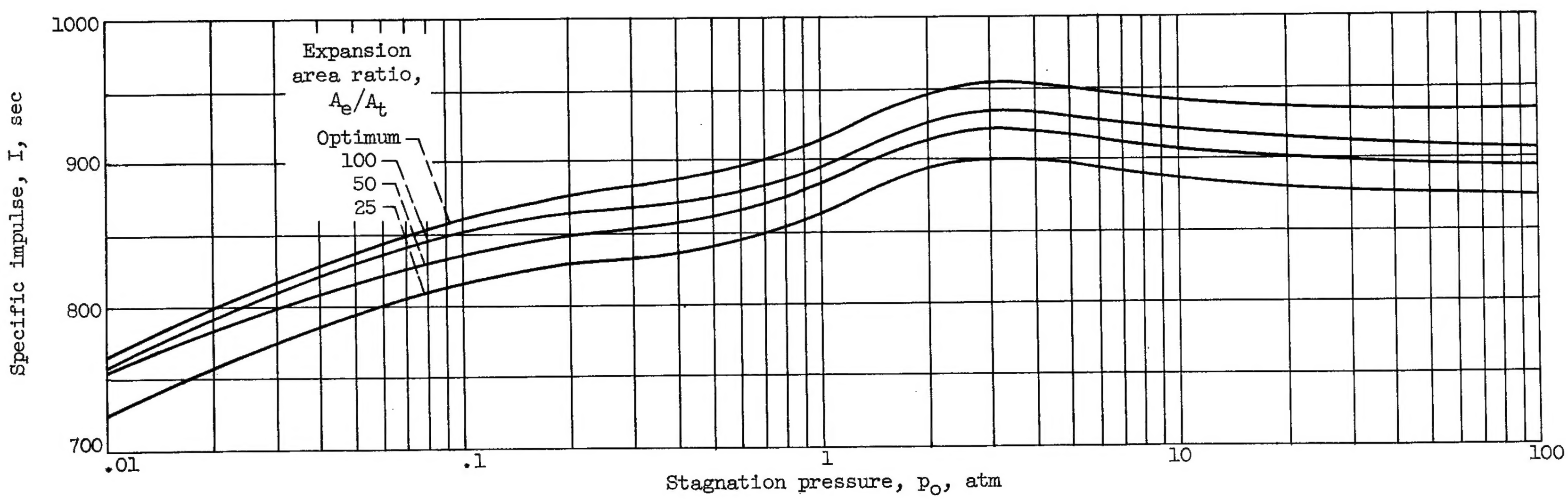


Figure 6. - Effect of stagnation pressure and nozzle geometry on specific impulse. Divergence angle, 15°.

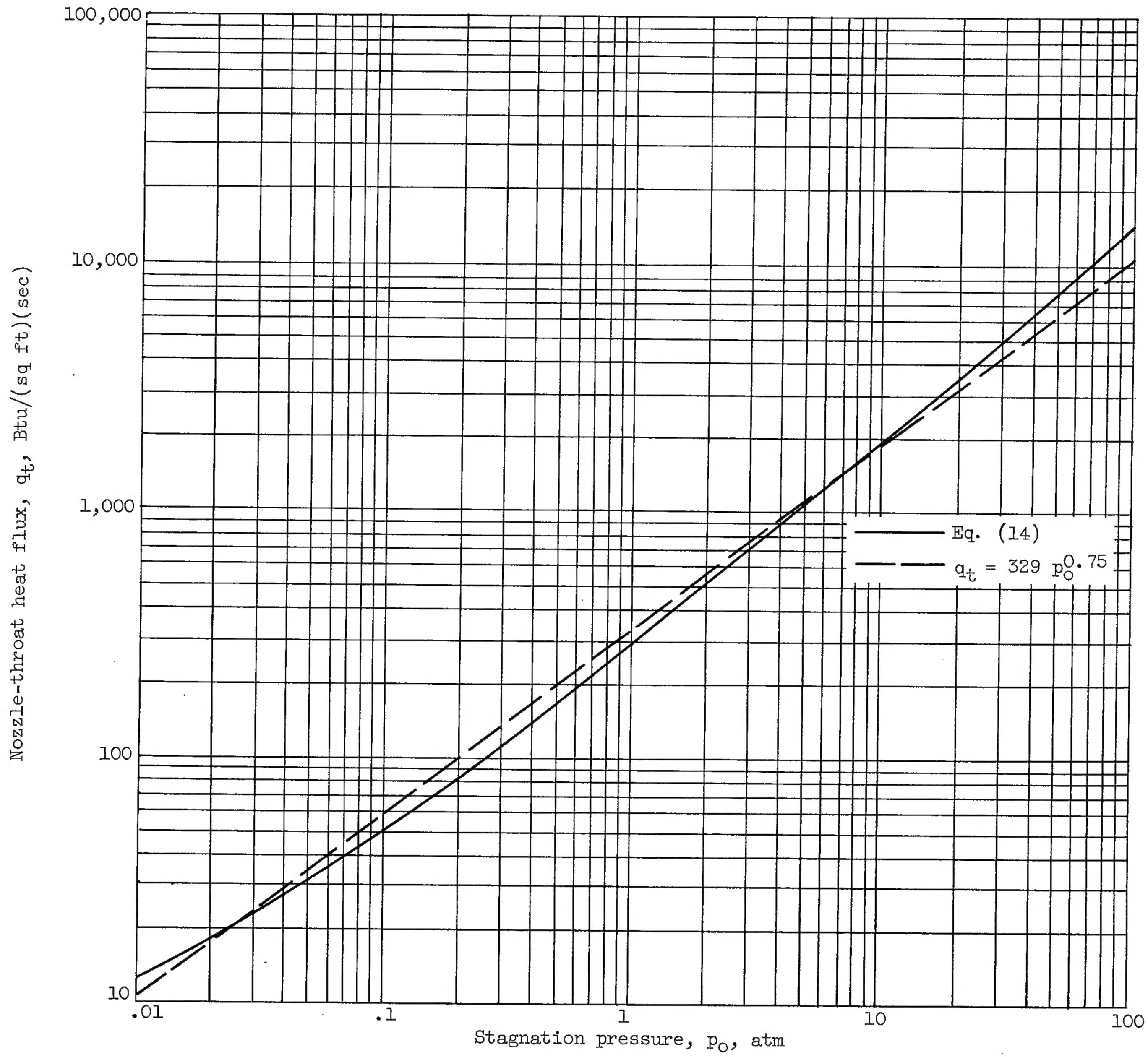


Figure 7. - Nozzle-throat heat flux.

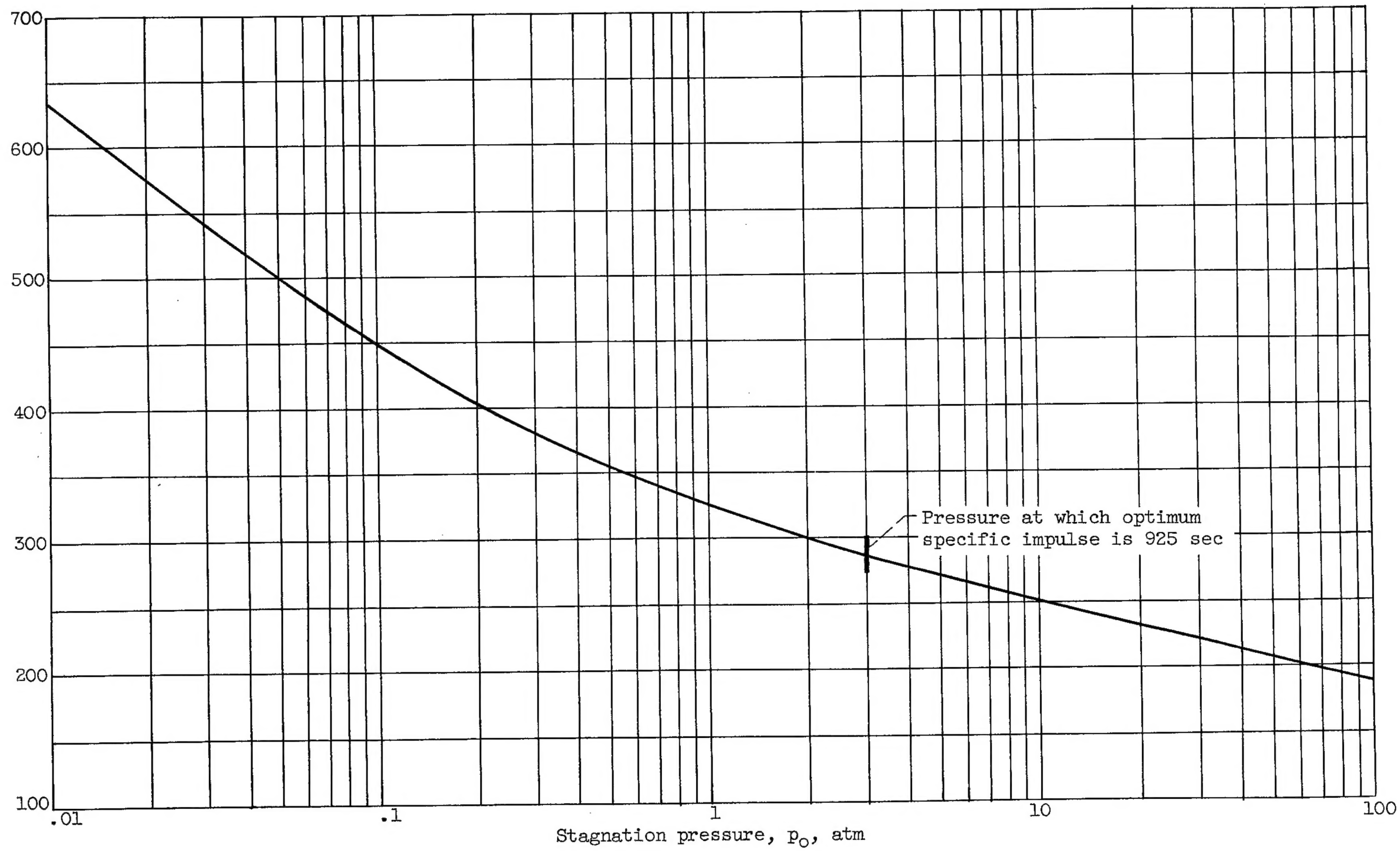


Figure 8. - Total nozzle heat load. Divergence angle, 15°; expansion area ratio, 50.

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Conclusions were drawn concerning (1) optimum values of nozzle area ratio and operating pressure and (2) operating conditions for favorable nozzle heat transfer. In general, the nozzle efficiency increases with both increasing divergence angle and increasing stagnation pressure level. For a given set of conditions, however, the nozzle efficiency increases with increasing area ratio only until the gains in performance due to additional expansion are offset by the increase in convective heat transfer. The variation of specific impulse with pressure is maximized at a pressure level that depends upon the pressure at which the flow freezes. On the basis of heat transfer, operation on the low-pressure side of the optimum point is desirable because a substantial reduction in the throat heat flux is thereby permitted.

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